# **Abbey Village primary School**

Mathematical Fluency Policy



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#### **Mathematical Fluency Policy**

#### Fluency involves;

- Quick recall of facts and procedures
- The flexibility and fluidity to move between different contexts and representations of mathematics.
- The ability to recognise relationships and make connections in mathematics



**Fluency** is one of the 'Five Big Ideas'. These are principles drawn from research evidence that underpin a 'Teaching for Mastery' approach. Fluency goes hand-in hand with the other ideas that lie at the heart of maths mastery pedagogy.

A child who is fluent in key maths facts has the ability to quickly and efficiently recall facts and procedures and has the flexibility to move between different contexts and representations of mathematics.

At Abbey Village primary School there is an emphasis on the importance of developing fluency with mathematical facts. Mathematics lessons begin with a fluency activity. Children are also given regular opportunities within and outside of maths lessons to practise basic facts and develop flexibility with these facts.

A CPA (concrete – pictorial – abstract) approach is followed which supports the development of fluency with key concepts. A number of concrete, pictorial and other resources are used to develop the understanding of basic facts and help children to become fluent in basic maths facts. Children may use the following resources to help secure fluency with number facts: stem sentences, Numicon, ten frames, bar models, part-whole models, counters and Dienes apparatus. Children develop their understanding of basic facts with concrete resources first before moving on to representing numbers and facts pictorially and then abstractly. When fluency with a fact develops children will no longer need resources and will be able to automatically recall that fact within three

seconds. At Abbey Village Primary there is a drive on fluency throughout school. This is being supported by an emphasis on concrete and pictorial representations of number in the Early Years and Year One.

- **1.** <u>Developing fluency in addition and subtraction facts -</u>*Why focus on fluency in addition and subtraction facts?*
- A defined set of addition and subtraction facts build the basis of all additive calculation, just as times tables are the building blocks for all multiplicative calculation:





Informal/mental addition by partitioning:

Root addition facts

3 + 4, 6 + 5

Formal subtraction with column method

Root subtraction facts

12 - 4, 5 - 2, 3 - 1

- If children are not fluent in these facts, then when they are solving more complex problems the working memory is taken up by calculating basic facts, and children have less working memory to focus on solving the actual problem so fluency in basic facts allows children to tackle more complex maths more effectively.
- Fluency is one of the 3 aims of the national curriculum, and external tests focus heavily on fluency.
- Children need to be taught strategies to solve these facts. If children aren't explicitly taught to solve e.g. 6+7 by thinking 'double 6 and one more' or to solve 12 8 by thinking '2 more and 2 more again' then most children will use inefficient counting-based approaches.

# What facts do children need to be fluent in?

Children need to be fluent in the following addition facts:

+	0	I	2	3	4	5	6	7	8	9	10
0	0 + 0	0 + 1	0 + 2	0 + 3	0 + 4	0 + 5	0 + 6	0 + 7	0 + 8	0 + 9	0 + 10
I	+ 0	+	1 + 2	+ 3	+ 4	+ 5	+ 6	1 + 7	+ 8	+ 9	1 + 10
2	2 + 0	2 + 1	2 + 2	2 + 3	2 + 4	2 + 5	2 + 6	2 + 7	2 + 8	2 + 9	2 + 10
3	3 + 0	3 +	3 + 2	3 + 3	3 + 4	3 + 5	3 + 6	3 + 7	3 + 8	3 + 9	3 + 10
4	4 + 0	4 + 1	4 + 2	4 + 3	4 + 4	4 + 5	4 + 6	4 + 7	4 + 8	4 + 9	4 + 10
5	5 + 0	5 + 1	5 + 2	5 + 3	5 + 4	5 + 5	5 + 6	5 + 7	5 + 8	5 + 9	5 + 10
6	6 + 0	6 + 1	6 + 2	6 + 3	6 + 4	6 + 5	6 + 6	6 + 7	6 + 8	6 + 9	6 + 10
7	7 + 0	7 + 1	7 + 2	7 + 3	7 + 4	7 + 5	7 + 6	7 + 7	7 + 8	7 + 9	7 + 10
8	8 + 0	8 + 1	8 + 2	8 + 3	8 + 4	8 + 5	8 + 6	8 + 7	8 + 8	8 + 9	8 + 10
9	9 + 0	9 + 1	9 + 2	9 + 3	9 + 4	9 + 5	9 + 6	9 + 7	9 + 8	9 + 9	9 + 10
10	10 + 0	10 + 1	10 + 2	10 + 3	10 + 4	10 + 5	10 + 6	10 + 7	10 + 8	10 + 9	10 + 10



T h

#### ese are the corresponding subtraction

facts:

-	0	I	2	3	4	5	6	7	8	9	10
Ι	1 – 0	1-1									
2	2 – 0	2 - 1	2 - 2								
3	3 - 0	3 - 1	3 - 2	3 - 3							
4	4 - 0	4 – 1	4 - 2	4 - 3	4 - 4						
5	5 - 0	5 - 1	5 – 2	5-3	5 – 4	5 - 5					
6	6 - 0	6-1	6 - 2	6 - 3	6 - 4	6 - 5	6 - 6				
7	7 – 0	7 - 1	7 - 2	7 - 3	7 - 4	7- 5	7 - 6	7 - 7			
8	8 – 0	8 - 1	8 – 2	8 - 3	8 – 4	8 -5	8 - 6	8 - 7	8 - 8		
9	9 – 0	9 - 1	9 – 2	9 - 3	9 – 4	9 - 5	9 - 6	9 - 7	9 - 8	9 - 9	
10	10 - 0	10 - 1	10 - 2	10 - 3	10 - 4	10 - 5	10 - 6	10 - 7	10 - 8	10 - 9	10 - 10
· · · · ·											
П		11+1	11 - 2	11 - 3	11-4	11 - 5	11 - 6	11 - 7	11 - 8	11 - 9	11 – 10
12			12 - 2	12 - 3	12 - 4	12 - 5	12 - 6	12 - 7	12 - 8	12 - 9	12 - 10
13				13 - 3	13 - 4	13 - 5	13 - 6	13 - 7	13 - 8	13 - 9	13 – 10
14					14 - 4	14 - 5	14 - 6	14 - 7	14 - 8	14 - 9	14 – 10
15						15 - 5	15 - 6	15 - 7	15 - 8	15 - 9	15 – 10
16							16 - 6	16 - 7	16 - 8	16 - 9	16 - 10
17								17 - 7	17 - 8	17 - 9	17 - 10
18									18 - 8	18 - 9	18 – 10
19										19 - 9	19 – 10
20											20 - 10

Note that not all subtractions within 20 are root facts, e.g. 17-5 is not considered a root fact (7-5 is the root fact for this).

The majority of these facts will be learnt in Year 1 and Year 2. In Reception, children become fluent in working with totals to 5 (though not recording as equations), e.g. "Show me 5 on your hands. Now show me 5 in a different way." Year 3 will need to focus on securing fluency in subtraction facts which bridge 10. Although this is a Year 2 objective, aiming for real fluency in subtraction facts such as 14 - 9 and 13 - 5 (where fluency is an answer in 3 seconds) requires securing in Year 3.

# Does fluency just mean memorisation?

Not necessarily – most rely on very quick use of strategies to solve some of them. Fluency can mean getting an answer quickly and with limited demands on working memory.

Most facts which don't bridge 10 are memorised, 4 + 5 = 9 or 2 + 6 = 8 for example.

For facts which bridge 10, the picture is more complex and many of the facts which bridge 10 are quickly derived using strategies (but still in less than 3 seconds).

- Double 6, 7 8 and 9 can be memorised in fluent children.
- Many fluent children may 'just know' that 9 + 3 = 12 and 8 + 4 = 12 and relate this to their times table/skip counting knowledge.
- Fluent children use strategies for many of the other facts. Eg 9 + 8 with fluency this can be solved through very quickly applying a strategy: bridging, near doubles or compensating.

The grid below demonstrates approaches taken by a fluent, high attaining Year 4 child to each of the addition facts: no counting approach was used for any of the facts, but they are not memorised either (K= Known fact; S= Strategy). The child attends a school within our Maths Hub.

	0	1	2	3	4	5	6	7	8	9	10	
0	К	К	К	К	К	К	К	К	К	К	К	Name MS
1	К	К	К	К	К	К	К	К	К	К	К	School AD
2	К	К	К	К	К	К	К	К	К	К	к	Year 4
3	К	К	К	К	К	К	К	К	S	S	к	Level 3a
4	К	К	К	К	К	К	К	S	К	S	К	
5	К	К	к	К	к	К	S	S	S	S	к	
6	К	К	К	К	К	S	к	S	S	S	к	Notes on strats
7	К	К	К	К	S	s	S	К	S	S	К	Predominantly bridging
8	К	К	к	S	к	S	S	S	К	S	К	Diff of 2 converted to doubles e.g. 6 + 8 saw as double 7
9	К	К	К	S	S	S	S	S	S	К	К	Strong commutativity - identical strats always used
10	К	к	к	K	K	K	K	K	K	K	к	

# How do children become fluent?

Children need to be **taught** strategies to derive the facts. Teaching strategies are more effective in securing fluency in addition and subtraction facts than taking a rote memorisation approach.

# Suggested progression

# Group A: Year 1 (Within 10)

- 1. Adding 1 (e.g. 7 + 1 and 1 + 7)
- 2. Doubles of numbers to 5 (e.g. 4 + 4)
- 3. Adding 2 (e.g. 4 + 2 and 2 + 4)
- 4. Number bonds to 10 (e.g. 8 + 2 and 2 + 8)
- 5. Adding 10 to a number (e.g. 5 + 10 and 10 + 5)
- 6. Adding 0 to a number (e.g. 3 + 0 and 0 + 3)
- 7. The ones without a family! 5 + 3, 3 + 5, 6 + 3, 3 + 6

Knowing these facts by the end of Year 1 will mean children will know 87 of the 121 addition facts in the grid.

# Group B: Year 2 (Bridging 10)

Children have 34 addition facts left to learn - they are the ones which bridge 10. While a few adults have instant recall of all of these, most rely on strategies for some. Our aim for children is that they use known facts or derived fact strategies to quickly recall or derive each fact. We need to ensure that all children move beyond counting based strategies. This will require careful teaching of the strategies combined with plenty of practice.

- 8. Doubles of numbers to 10 (e.g. 7 + 7)
- 9. Near doubles (e.g. 5 + 6 and 6 + 5)
- 10. Bridging (e.g. 8 + 4 and 4 + 8)
- 11. Compensating

Note that these 3 strategies can often be used interchangeably, e.g. for 8 + 9, some people will use near doubles (e.g. 8 + 8 + 1), some will use bridging (e.g. 8 + 2 + 7) and some will use compensating (8 + 10 - 1)

N.B. Before the children are ready to learn bridging as a strategy, they need to be able to partition all single digit numbers, therefore the following facts need to be taught alongside the above facts:

- Partitioning 2, 3, 4, 5, 6 and 10
- Partitioning 7, 8 and 9
- Partitioning 11 20 into single digit addends

Once children have been taught the strategies, they need to move on to **PRACTICE** of the facts. The aim is for an average of 3 seconds or less per fact.

Generally, for practice

- We focus on practising the set of facts being learnt (or just learnt) in isolation for a few days
- We focus on mixing these up with all previously learnt facts

For each of these 11 steps, a suggested teaching approach is laid out below, including manipulatives/images, key teaching points and a suggested teaching progression.

# Step 1: Adding 1 to a number



# **Key teaching points**

1<sup>st</sup> key point: Adding 1 to a number is the same as '1 more than' that number  $2^{nd}$  key point: Commutativity 7 + 1 = 1 + 7

#### **Teaching progression:**

**Concrete:** Use equipment and a numbered number line to be able to say what is 1 more than any number to 10.

Pictorial: Represent this knowledge in part-part-whole Diagrams

Abstract: Record this knowledge using number sentences; Model that these can be expressed **commutatively** 1+7=8 or 7+1=8





Model that these can also be expressed as **partitioning** the whole 8 = 1 + 7 or 8 = 7 + 1

# Step 2: Doubles of numbers to 5

	Images/manipulatives	
Numicon pieces		
Doubles written up	2 4 6 8 10	
	2 + 2 = 4 3 + 3 = 6	
	4 + 4 = 8 5 + 5 = 10	

# **Key teaching points**

1<sup>st</sup> key point: Our doubles of numbers to 5 are all even numbers [as appropriate you can lead children to the idea that doubling a whole number always gives us as even number] 2<sup>nd</sup> key point: We need to learn our doubles off by heart!

#### **Teaching progression:**

Awareness of odd and even: Be able to identify numbers as odd or even, using Numicon as a visual image

Fluency in odds and evens counting: Practice counting in even numbers Understanding of what doubles is: "Double 5" = "Two lots of 5" [spoken] = 5 + 5. Can the children show you these with Numicon pieces or fingers on each hand? Noticing patterns: Look as a class at the doubles pattern and relate to even numbers PRACTICE: Now you need to play LOTS of doubles games until the children all know their doubles of numbers to 5 off by heart. This is one of the sets which the children just need to memorise.

Represent in part-part whole models and in number sentences

# Step 3: Adding 2 to a number



#### **Key teaching points**

**1**<sup>st</sup> **key point:** When we add 2 to a number, we are working within our odds and evens counting pattern

 $2^{nd}$  key point: Commutativity 7 + 2 = 2 + 7

#### Possible teaching progression:

Awareness of odd and even: Be able to identify numbers as odd or even, using Numicon as a visual image

**Fluency in odds and evens counting:** Practice counting in odds and evens to 20, forwards and backwards until fluent. Use odd and even number lines for support.

**Concrete:** Use Numicon to see that when we add 2 to a number (or when we add a number to 2) we are just making the next odd/even number.

**Pictorial:** Represent this knowledge in part-part-whole Diagrams





**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole

# Step 4: Number bonds to 10

	Images/mani	oulatives	
Numicon pieces			Ň
Number bonds to 10 written	0 + 10 = 10	10+0=10	up
	+ 9 =  0	9 + 1 = 10	
	2 + 8 = 10	8 + 2 = 10	
	3 + 7 = 10	7 + 3 = 10	
	4 + 6 = 10	6 + 4 = 10	
	5 + 5 = 10		

# Key teaching points

**1**<sup>st</sup> **key point:** Our number bonds to 10 are always odd + odd OR even + even **2**<sup>nd</sup> **key point: Commutativity** 6 + 4 = 4 + 6**3**<sup>rd</sup> **key point:** We need to learn our number bonds to 10 off by heart!

#### Teaching progression:

Awareness of odd and even: Be able to identify numbers as odd or even, using Numicon as a visual image

**Exploring different ways of making up 10:** Using the Numicon for support, notice that the number bonds to 10 are always odd + odd or even + even

**PRACTICE:** Now you need to play LOTS of games until the children all know their number bonds to 10 off by heart. This is one of the sets which the children just need to memorise.

Represent in part-part whole models and in number sentences.

Pictorial: Represent this knowledge in part-part-whole diagrams

	Ö
8	2

**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partioning the whole

#### Step 5: Adding 10 to a number



#### Key teaching points

**1**<sup>st</sup> **key point:** When we add 10 to a number we can use our place value knowledge to combine the numbers

**2^{nd} key point: Commutativity** 10 + 5 = 5 + 10

# **Teaching progression:**

**Place value experience:** Make up 'teens' numbers with place value equipment e.g. straws (or Numicon/Dienes).

**Relate place value representation to notation:** *"This is the number fifteen. We write it 15 because there is one ten and five ones."* 

Pictorial: Represent this knowledge in part-part-whole diagrams

**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole

# 15 10 5

# Step 6: Adding 0 to a number

Images/manipulatives

Counters/straws/Numicon would all do here.

# Key teaching points

**1**<sup>st</sup> **key point:** When we add 0 to a number we are adding nothing, and so our starting number remains the same. [Misconception here is that 7 + 0 = 0] **2**<sup>nd</sup> **key point: Commutativity** 0 + 4 = 4 + 0

#### **Teaching progression:**

**Practical experience of making up number sentences involving 0:** Show me 0. Now add 4. How much do you have? Show me 4. Now add 0. How much do you have?

Stem sentence: "When we add 0, we don't change the quantity."

Pictorial: Represent this knowledge in part-part-whole diagrams

**Abstract:** Record this knowledge using number sentences; Model that these can be expressed commutatively and by partitioning the whole

#### Step 7: The ones without a family

The only remaining Y1 facts are 6 + 3 & 3 + 6 and 3 + 5 & 5 + 3. These just need to be learnt. Fluent children often relate 6 + 3 to the counting in 3s pattern.

For 5 + 3 and 3 + 5 (indeed for any addition fact involving 5) children can be taught to recognise the standard "finger pattern" for 8 of 5 fingers and 3 fingers fairly easily, then this can be related to 5 + 3 and vice versa (incidentally it is worth getting all reception children to recognise 6, 7, 8, and 9 when presented in this way, then they already 'know' 5 + 1, 5 + 2, and 5 + 4 as well (they just need to be *taught* that they already know them!).



# Step 8: Double 6, 7. 8 and 9



#### Images/manipulatives

Double sided counters can model double 6, 7, 8 and 9 as double "5 and a bit" (i.e. double 8 is double 5 add double 3)

Numicon will allow the children to see that doubles of whole numbers are always even numbers

Key teaching points 1<sup>st</sup> key point: Doubles of whole numbers are always even 2<sup>nd</sup> key point: We need to learn our doubles off by heart!

#### Teaching progression:

[From Y1, children should be able to identify even numbers and know that a double means two lots of]

Teach as follows:

**Double 6:** use the clock face. 6 at the bottom, 12 at the top.

**Double 7:** explain that two weeks is called a fortnight because it has 14 nights. There are 7 days in a week, so double 7 is 14.

**Double 8 & double 9**: for a few children, remembering which is 16 and which is 18 seems particularly hard. There isn't any substitute for practice here. Keep asking any target children this many times each day for a week, and keep a record of which children don't yet know it. **Relating to inverse.** What is half of 14 etc.

Once the facts are learnt, represent in part-part whole and equations as before.

#### Step 9: Near doubles

6 + 7 = 6 + 6 + 1

# Images/manipulatives

Adjacent numbers recognised as being 'near doubles' and 'one up one down' (i.e. second model shown here) is also a really nice efficient use of doubling.

#### **Key teaching points**

**1**<sup>st</sup> **key point:** I can add adjacent numbers by doing 'double and 1 more' **2**<sup>nd</sup> **key point:** I can add number with a difference of 2 (e.g. 6 + 8) by doubling the number in between them (i.e. by doubling 7 in this case) **3**<sup>rd</sup> **key point:** Commutativity: 5 + 6 = 6 + 5

#### Teaching progression:

Fluency in doubles: Will already have been secured

6 + 8 = 7 + 7

Adjacent numbers: Will be double the smaller number, add 1. OR double the larger number, subtract 1.

**Difference of 2:**5 + 7, 6 + 8, 7 + 9

Then part-part whole and practice with equations as before

# Step 10: Bridging



#### Key teaching points

**1**<sup>st</sup> **key point:** Bridging through ten can help us to calculate additions with a 'teens' total **2**<sup>nd</sup> **key point:** Commutativity: 5 + 8 = 8 + 5

#### Teaching progression:

 Partitioning single digit numbers: Children HAVE to be able to do this to bridge.

 Calculating e.g. 8 + 5 by bridging requires partitioning the 5 into 2 and 3

 What makes ten?: Children need to be able to make ten from 7, 8 and 9 (which are most likely to be involved in bridging facts)

 Tens frames (concrete): Make up the two quantities with couters on adjacent tens frames, then rearrange as shown above.

 Symbolic: Practice recording as number sentences (as shown above)

 Part-Part Whole: Move to filling in PPW as shown here:

 8

 yresented as number sentences by thinking "8 + 2 + 3" in their heads

 Comparison to other strategies: Highlight that we can also use e.g. near doubles to solve some bridging facts (e.g. 8 + 7)

# Step 11: Compensating/adjusting

Images/manipulatives The children should already be fluent in e.g. 5 + 10 and 10 + 5 5 + 10 = 15 so 5 + 9 = 14

#### Key teaching points

**1**<sup>st</sup> **key point:** By subtracting one from 'add ten' I get 'add nine' **2**<sup>nd</sup> **key point:** Commutativity (5 + 9 = 9 + 5)

#### Teaching progression

Fluency in adding ten: will already have been secured
Then PPW and practice with number sentences as before
Adding 8 and 7: Highlighting possibility of using compensating for adding numbers other than 9 (e.g. 8 and 7)

**Comparison to other strategies**: Highlight that we can also use near doubles and briding to solve some compensating facts, e.g. 8 + 9

#### 2. Developing fluency in multiplication and division facts

Children at Abbey Village primary are provided with regular opportunities, both in class and through engaging homework activities, to develop times tables knowledge. This ensures rapid recall of multiplication and division facts. Children in key Stage Two experience times tables tests which show questions and numbers arranged in a variety of ways, using pictorial representations of numbers and missing number questions to develop flexibility and fluency with these facts. Using tests that require children to think in a variety of ways about times tables, alongside the traditional recall of facts style of testing, ensures children develop depth of understanding and the flexibility to apply multiplication and division knowledge to unfamiliar contexts. See appendix 1 for examples of tests which promote depth of thinking with times tables facts.

Times tables songs and 'Times Tables Rockstars' are used to support the learning of times tables.

Fluency in times tables at Abbey Village primary is reinforced through use of the very popular Times Tables Rock Stars programme, either on screen or through paper copies. Children have the opportunity to use this at school and are encouraged to use it at home as daily times tables practice. The programme is used in classes in a competitive way and the children find the programme fun, engaging and motivating to use.



Fluency is further developed in Key Stage 1 and 2 with the use of our 99 club challenges and in Upper Key Stage Two with the use of Numeracy Ninjas. Year 5 and Year 6 enjoy working through the daily 5 minutes skills tests, trying to beat their own previous Ninja scores and collecting Ninja belts (see appendix 2). Certificates are awarded to the children who achieve 'Grand Master' status (this can be given by the teachers to reward consistent high achievement and also improved performance/ good effort). See appendix 3.



Pupils in Upper Key Stage 2 who need to revisit specific skills can use the intervention resources on the Numeracy Ninjas website (Ninja Skill Focus Worksheets - Numeracy Ninjas) for further practice. See appendix 4 for an example of these.

Example of 5 minute fluency skills practice session



Name:

3+3=	7+7=
2+2=	1+1=
5+5=	8+8=
4+4=	9+9=
6+6=	10+10=
0+0=	



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N U M E R A C Y N I N J A S . O R G









# THIS CERTIFICATE IS PROUDLY PRESENTED TO

# WHO ACHIEVED GRAND MASTER STATUS





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# Number Bonds To 10 Mental Strategies

Complete the daily exercises to focus on improving this skill.

Day	Day 1							
σ	Question	Answer						
1	8 + 2 = 🗆							
2	6 + 🗆 = 10							
3	□ + 7 = 10							
4	6 + 4 = 🗆							
5	2 + 🗆 = 10							
6	10 + 🗆 = 10							
7	6 + 🗆 = 10							
8	8 + 🗆 = 10							
9	8 + 🗆 = 10							
10	10 + 🗆 = 10							

Day	Day 2						
σ	Question	Answer					
1	1 + 🗆 = 10						
2	7 + 🗆 = 10						
3	□ + 9 = 10						
4	3 + 🗆 = 10						
5	10 + 🗆 = 10						
6	4 + 🗆 = 10						
7	□ + 2 = 10						
8	6 + 4 = 🗆						
9	□ + 10 = 10						
10	□ + 2 = 10						

Day	Day 3						
α	Question	Answer					
1	2 + 🗆 = 10						
2	□ + 1 = 10						
3	□ + 1 = 10						
4	3 + 7 = 🗆						
5	2 + 🗆 = 10						
6	□ + 8 = 10						
7	6 + 🗆 = 10						
8	1 + 🗆 = 10						
9	2 + 8 = 🗆						
10	10 + 🗆 = 10						

Day	Day 4						
α	Question	Answer					
1	□ + 3 = 10						
2	4 + 🗆 = 10						
3	□ + 2 = 10						
4	9 + 1 = 🗆						
5	□ + 7 = 10						
6	□ + 4 = 10						
7	3 + 🗆 = 10						
8	2 + 🗆 = 10						
9	□ + 7 = 10						
10	1 + 🗆 = 10						

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